Mathematics As an Artistic-Generative Principle

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ABSTRACT

The author defines a mathematical discipline that is devoted to the generation of artistic images. The practical implementation of the underlying theory is possible today with the aid of computer graphics systems.

Interconnections between mathematics and art are manifested for the most part in the form of principles of order that can be observed in works of art. Especially in the works of classical art, symmetry was explored by Hermann Weyl [1]; he used mathematical methods for analysis and description of various types of artwork. Up until now other mathematical characteristics of artistic pictures—for example those of combinatorics (Karl Gerstner, Richard Paul Lohse, Shizuko Yoshikawa), of the theory of numbers (Rune Mields, Anton Stankowski), of aleatorics (Gerd von Graevenitz, Herman de Vries), and others were stated but not considered more deeply from the point of view of general theoretical relations [2].

A further area of common ground can be seen in the fact that the visualization of mathematical relationships often leads to aesthetically pleasing results. A good illustration of this is the computer graphics work with fractals begun by Benoit B. Mandelbrot and continued by the Bremen-based "Working Group on Complex Dynamics" headed by Heinz-Otto Peitgen and Peter H. Richter [3]. In these and similar cases, the beauty of the pictures obtained is regarded as a pleasant side effect that can be enhanced further by selection of appropriate sections and colors. This represents an initial, still hesitant step towards artistic creativity; it yields results which can still be treated as mathematical documents in pictorial form, but for which its practitioners claim artistic value, as is expressed in the term 'map art' coined by the Bremen working group. Visually attractive pictures can also be achieved using other fields of mathematics, such as field theory, the theory of complex functions, Fourier transformations [4] and—most recently—topology [5]. In all these cases the fascinating visual results are more a subordinate effect, but they prove that this method shows a yet-widely untouched field of interesting forms and shapes. Enormous possibilities lie in mathematical research conducted with artistic rather than mathematical goals.

In the past, there have been only few indications in this direction; the most important work—a systematic investigation in the fields of algebra and analysis—was conducted by Maurice El-Milick in 1936 [6]. Also remarkable are the publications of Hermann von Baravalle [7], who devoted his work to a series of geometrical shapes.

The development of artistic computer graphics implies a strong impulse to use sophisticated mathematical relations for artistic creations. In particular, those working with mathematical plotters—from Frieder Nake, Georg Nees and A. Michael Noll to Collette and Jeff Bangert, Harold Cohen and Edward Zajec [8]—used mathematics in pretentious fashion to produce new structures, unknown in art up to this time; but unfortunately, they did not theoretically systematise their methods. On the other side, utilisation of menu-controlled computer-aided design (CAD) and paint systems is a step backward to conventional picturemaking and a corresponding renunciation of innovations coming from mathematics in this field. There are, however, some artists using sophisticated mathematics—for example Jeffrey Ventrella uses fractals [9] while Donna Cox employs physical research with supercomputers [10].

As a logical consequence of these developments and in order to clarify the situation, it is appropriate at this point to name and define the procedure that underlies these attempts: *Generative mathematics is defined as the study of mathematical operations suitable for generating artistic images.*

Following are some suggestions for some of these goals:

(1) One of the focuses of generative mathematics is the derivation of functions that, when graphically displayed and viewed, yield aesthetically interesting results. These can be either graphic elements suitable for composing images, or configurations that can be used as the basis for further processing. The functional relations that deserve particular attention are not those that are the expression of any scientific discoveries (perhaps for this very reason have been ignored in the past), but those that bring forth new and fascinatingly beautiful forms.

Yet another fundamental activity of generative mathematics is the development of transformations that can be applied to images; no longer of relevance here are the mathematical attributes on which interest previously focused, but rather the possibilities they offer for aesthetic optimization.

(2) One important concern in the field of generative mathematics is the concern with the description of image structures, a process analogous to the transcription of music. This line of study concentrates on aspects of mathematical formalism and on the algorithms used by the computer programs.

(3) An interesting aspect of generative mathematics is its link with rational aesthetics. This has to do, for example, with the extent to which mathematically expressible

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principles can be correlated with aesthetic effects. Quantifiable image description also provides a useful basis for statistical studies of the aesthetics of information.

(4) Historical questions (particularly those belonging to the recent past) are relevant to generative mathematics. For example, it would be desirable to compile an inventory of all of the computer graphics methods that have yielded aesthetically interesting results during the last 25 years. These include the use of random number generators, which has been practiced right from the start and still plays a significant role in connection with depiction of fractal geometry. A collection of earlier publications dealing with connections between mathematics and art would also be very useful.

(5) Finally, generative mathematics also includes procedural issues related to the hardware and software problems associated with computer graphics. For example, the method of 'experimental mathematics' that is currently being discussed avidly in specialist publications can be readily tied in with aesthetic issues. It would also be worthwhile to discuss questions related to instruments and methods in connection with the concept of the 'aesthetic laboratory' introduced by Georg Ness.

CONCLUSIONS

Many artists are opposed to theoretical considerations such as those discussed here. The question arises: Do aesthetic experiments with mathematical relations need in fact a theory? I think the answer is yes, in order not to overlook all the possibilities and consequences in the utilisation of complicated mathematical methods and their application in a highly sophisticated tool, the computer. It is not simply a question of how to produce beautiful pictures; in this context, the question arises as to whether generative mathematics could not also open up new possibilities for representation and expression. A parallel to this can be found in music, where the development of instruments combined with the theory of harmony has given rise to an extraordinary artistic development.

The use of instruments and other aids results in new techniques, and new techniques in turn inspire new ways of thinking. For example, the mathematical method of image generation leads to a departure from the classical way of composing a picture. In the conventional approach, the picture is changed only where the artist directly intervenes (with a brush or pencil, for example). This approach can therefore be described as 'punctual'. The advent of a mathematical way of thinking, by contrast, makes available the possibility of altering the entire picture with each intervention (e.g. by means of a transformation). This approach can thus be characterized as 'integral'.

In principle, any image can be constructed using either of these two approaches, but apparently certain visual structures exist that lend themselves more readily to implementation by the punctual method and others that are better suited for integral composition. In the past, those in the latter category have all too often been neglected—predominantly those that do not consist of arbitrarily placed elements, but instead obey a uniform principle (even though this may be a complex one). In this respect, they are more closely related to music than to the classical visual arts.

The rapid switch from one image to another made possible by computer graphics systems facilitates the transition from still images to moving pictures. In other words, the mathematical method is, in a manner of speaking, inherently suited for depiction of dynamic processes. This yields interesting possibilities for putting into practice the old idea of a dynamic play of graphic forms. If this can be described with the aid of mathematical formulas, then it is particularly easy to implement it with a computer program. Even the idea of 'graphic improvisation' in realtime can be achieved in this way without difficulty.

It is of course also possible to use mathematical methods for image generation without having a theoretical background—as has already been shown by the work of various individuals. But if the aim is to exploit the gigantic potential that exists for the creation of mathematically describable images, then 'generative mathematics' is capable of making a vital contribution.

References and Notes

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